DETERMINATION OF THE COEFFICIENT OF INTERNAL HEAT EXCHANGE AND THE EFFECTIVE THERMAL CONDUCTIVITY OF A POROUS SOLID ON THE BASIS OF A NONSTATIONARY EXPERIMENT

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We construct an iterative algorithm for determining the coefficient of internal heat exchange and the effective thermal conductivity of a porous solid on the basis of data obtained in temperature measurements.

Calculating the thermal regime of porous structural elements subjected to the action of powerful surface or internal heat sources requires above everything else a knowledge of the laws governing the heat exchange on the external surface and inside of the porous structure.

Experimental studies of heat exchange inside porous materials have been carried out in many published works, for example [1-7]. However, as was noted in [7-9], the results obtained in such studies can be used only after further experimental verification under conditions corresponding to the real situation, since there is a considerable observed dispersion in the experimental data of different authors; this dispersion is due, among other factors, to the difference between the methods used to determine the volumetric heat-exchange coefficient. The practical importance of this problem means that further investigations in this inadequately studied area of heat exchange must be conducted, giving special attention to making sure that the methods of conducting the experiments and processing their results are correct.

The intensive development of the theory and methods of solution of inverse problems in heat exchange has led to a considerable expansion of the area of their practical application in thermophysical investigations. This principle can also serve as the basis for investigating heat exchange in the case of porous cooling [10]. An important class of such problems consists of inverse coefficient problems, in which, within the framework of the selected mathematical model of the heat-exchange process and with given boundary conditions, we are required to find the thermophysical characteristics from the results of temperature measurements at several points of the specimen.

In the present study we consider an inverse problem in the determination of the coefficient of internal heat exchange and the effective thermal conductivity of a porous skeleton. The results of the investigations carried out by different authors indicate that among the quantities determining the value of the coefficient of internal heat exchange, the most important ones are the characteristics of the porous medium, the thermophysical properties of the coolant, and the intensity with which it is being blown through [7]. Only the blowing intensity can be varied during the experiment. The effective thermal conductivity of the porous skeleton is most affected by the characteristics and the temperature of the porous medium [11]. On the basis of these investigations, we shall try to find the coefficient of internal heat exchange as a function of time and to find the effective thermal conductivity, which takes account of the transfer of heat in the porous skeleton by conduction and radiation, as a function of the solid-phase temperature. If we obtain these functions in such form, we can divide the solution of the overall problem into two independent inverse coefficient problems, a fact which is especially important when we have one thermocouple on a porous plate. If several thermocouples are set up across the porous plate. It is possible to determine the two desired functions simultaneously. We shall consider this general case below.

It is required to find the vector function $\{T_s(x, \tau), \lambda_s(T_s), \alpha_V(\tau)\}$ from known nonstationary measurements of the temperature of the porous skeleton, knowing the initial distribution of the temperatures for the solid and gaseous phases, the law of variation of the coolant flow rate as a function of time, the hydraulic characteristics of the porous plate, and the varia-

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tion of the thermophysical characteristics of the blown-through gas and of the volumetric heat capacity of the porous solid as functions of the corresponding temperature. We shall assume that the process of heat exchange in the porous solid can be described by one-dimensional nonstationary differential equations of heat propagation in the porous skeleton and the coolant, whose temperatures differ from each other, and as the boundary conditions for calculating the thermal regime of the porous unbounded plane plate $0 \leqslant x \leqslant b$ we use boundary conditions of the second kind on its outer surface and of the third kind on its inner surface.

The mathematical statement of the problem so formulated is the following:

$$C_s \frac{\partial T_s}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_s \frac{\partial T_s}{\partial x}\right) - \frac{\alpha_V(\tau)}{1-\Pi}(T_s - T_g) + q_V; \tag{1}$$

$$\rho C_{pg} \frac{\partial T_g}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_g \frac{\partial T_g}{\partial x}\right) - \rho v C_{pg} \frac{\partial T_g}{\partial x} + \frac{\alpha_V(\tau)}{\Pi}(T_s - T_g), \tag{2}$$

$$0 < x < b, \quad 0 < \tau \leqslant \tau_m;$$

$$-\lambda_s \frac{\partial T_s(0,\ \tau)}{\partial x} = -\alpha_0 (T_s(0,\ \tau) - T_{g0}); \tag{3}$$

$$\rho v C_{pg} T_g(0,\ \tau) = \rho v C_{pg} T_{g0} + \alpha_0 (T_s(0,\ \tau) - T_{g0}); \tag{4}$$

$$-\lambda_s \frac{\partial T_s(b,\ \tau)}{\partial x} = q_w(\tau); \tag{5}$$

$$\frac{\partial^2 T_g(b,\ \tau)}{\partial x^2} = 0; \tag{6}$$

$$T_s(x,\ 0) = \xi_s(x), \quad 0 \leqslant x \leqslant b; \tag{7}$$

$$T_g(x,\ 0) = \xi_g(x), \quad 0 \leqslant x \leqslant b; \tag{8}$$

$$T_s(x_i,\ \tau) = f_i(\tau), \quad 0 \leqslant x_i \leqslant b. \tag{9}$$

The additional conditions are:

the equation of state of the gas

$$p = \rho \frac{\bar{R}}{M} T_g, \tag{10}$$

and the modified Darcy law

$$-\frac{dp}{dx} = \alpha \mu_g v + \beta \rho v^2. \tag{11}$$

The functions $q_w(\tau)$, $\rho v(\tau)$, $f_i(\tau)$, $\xi_s(x)$, $\xi_g(x)$, the porosity Π, the hydraulic characteristics of the porous solid, and the thermophysical characteristics of the coolant are assumed to be given.

With regard to the condition (6) used in this formulation, we must give an explanation. The difficulty is that it is often impossible to measure the temperature of the gas directly or to specify the density of the heat flux entering the gas at the external boundary of the porous solid. Data obtained by numerical simulation have shown that the calculated results found by using the free boundary condition (6) are in good agreement with the results corresponding to the specification of actual boundary conditions of the first and second kinds.

Since the stability of the inverse problem formulated above cannot be guaranteed in the general case [12], its solution must be regularized in some way. For this purpose we use iterative regularization, based on the algorithm of conjugate gradients, with the condition that the approximation is stopped according to the discrepancy criterion.

For the inverse problem (1)-(9) we formulate an extremel statement in the following form: we shall seek the vector function **R** = $\mathbf{R}(\alpha_V(\tau),\ \lambda_s(T_s))$ that will minimize the mean-squre discrepancy

$$J(\alpha_V,\ \lambda_s)=\sum_{i=1}^{n}\int_{0}^{\tau_m}[T_s(x_i,\ \tau)-f_i(\tau)]^2\,d\tau \tag{12}$$

under the conditions (1)-(8). When speaking of the minimization functional (12), we shall assume that it is minimized with due regard for the limitation of the permissible level of discrepancy, which is determined by the errors of the initial data.

In order to calculate the gradient of the functional (12), which is needed when the gradient method is used for solving the inverse problem, we shall use the solution of a boundary-value problem which is conjugate to the problem when an increment is added to the temperature fields in the solid and the gas.

Regarding the problem (1)-(9) so formulated as a multilayer problem, where the boundaries of layers having identical thermophysical properties coincide with the points of insertion of the thermocouples, we obtain a formula for the components of the gradient of the functional with respect to the desired parameters. We shall assume that there is "ideal" contact between the layers and that the contact thermal resistances are zero, i.e., the following conditions are satisfied:

$$T_{si}(x_{i+1},\ \tau)=T_{si+1}(x_{i+1},\ \tau),$$

$$\frac{\partial T_{si}(x_{i+1},\ \tau)}{\partial x}=\frac{\partial T_{si+1}(x_{i+1},\ \tau)}{\partial x}.$$

We assume that the components of the desired vector **R** have been given small increments $\Delta\alpha_V$ and $\Delta\lambda_s$, respectively. The temperatures T_{si} and T_g are given small increments $z_i(x,\ \tau)$ and $u(x,\ \tau)$ satisfying in the linear approximation the boundary-value problem:

$$\frac{\partial C_s z_i}{\partial\tau}=\frac{\partial^2(\lambda_s z_i)}{\partial x^2}+\frac{\alpha_V}{1-\Pi}(u-z_i)-\frac{\Delta\alpha_V+u\dfrac{\partial\alpha_V}{\partial T_g}}{1-\Pi}(T_{si}-T_g)+\frac{\partial}{\partial x}\left(\Delta\lambda_s\frac{\partial T_s}{\partial x}\right); \tag{13}$$

$$\frac{\partial\rho C_{pg}u}{\partial\tau}=\frac{\partial^2(\lambda_g u)}{\partial x^2}-\rho v\frac{\partial C_{pg}u}{\partial x}-\frac{\alpha_V}{\Pi}(u-z_i)+\frac{\Delta\alpha_V+u\dfrac{\partial\alpha_V}{\partial T_g}}{\Pi}(T_{si}-T_g),\ x_i<x<x_{i+1}, \tag{14}$$

$$0<\tau\leqslant\tau_m,\ i=1,\ 2,\ \ldots,\ n-1;$$

$$z_i(x,\ 0)=0; \tag{15}$$

$$u(x,\ 0)=0; \tag{16}$$

$$z_1(0,\ \tau)\frac{\partial\lambda_s}{\partial T_s}\frac{\partial T_{s1}(0,\ \tau)}{\partial x}+\lambda_s\frac{\partial z_1(0,\ \tau)}{\partial x}+\Delta\lambda_s\frac{\partial T_{s1}(0,\ \tau)}{\partial x}=\alpha_0 z_1(0,\ \tau); \tag{17}$$

$$\rho v\left(C_{pg}+\frac{\partial C_{pg}}{\partial T_g}T_g(0,\ \tau)\right)u(0,\ \tau)=\alpha_0 z_1(0,\ \tau); \tag{18}$$

$$\lambda_s(b,\ \tau)\frac{\partial z_{n-1}}{\partial x}+\Delta\lambda_s\frac{\partial T_{sn-1}(b,\tau)}{\partial x}+z_{n-1}\frac{\partial\lambda_s}{\partial T_s}\frac{\partial T_{sn-1}(b,\ \tau)}{\partial x}=0; \tag{19}$$

$$\frac{\partial^2 u(b,\ \tau)}{\partial x^2}=0, \tag{20}$$

where at the junctions between the layers the following conditions are satisfied:

$$\frac{\partial z_i(x_{i+1},\ \tau)}{\partial x}=\frac{\partial z_{i+1}(x_{i+1},\ \tau)}{\partial x},\ i=1,\ 2,\ \ldots,\ n-2; \tag{21}$$

$$z_i(x_{i+1}, \tau) = z_{i+1}(x_{i+1}, \tau). \tag{22}$$

For the linear part of the increment of the target functional (12) we have

$$\Delta J(\Delta\alpha_V, \Delta\lambda_s) = 2 \sum_{i=1}^{n} \int_0^{\tau_m} [T_s(x_i, \tau) - f_i(\tau)]\, z_i(x_i, \tau)\, d\tau. \tag{23}$$

We now consider the boundary-value problem conjugate to the system (13)-(20):

$$-C_s \frac{\partial \psi_i}{\partial \tau} = \lambda_s \frac{\partial^2 \psi_i}{\partial x^2} - \alpha_V \left(\frac{\psi_i}{1 - \Pi} - \frac{\varphi}{\Pi} \right); \tag{24}$$

$$-\rho C_{pg} \frac{\partial \varphi}{\partial \tau} = \lambda_g \frac{\partial^2 \varphi}{\partial x^2} + \rho v C_{pg} \frac{\partial \varphi}{\partial x} + \left(\frac{\psi_i}{1 - \Pi} - \frac{\varphi}{\Pi} \right) \left(\alpha_V - \frac{\partial \alpha_V}{\partial T_g} (T_{si} - T_g) \right), \; x_i \leqslant x \leqslant x_{i+1}, \tag{25}$$

$$0 \leqslant \tau < \tau_m, \; i = 1, 2, \ldots, n-1;$$

$$\psi_i(x, \tau_m) = \varphi(x, \tau_m) = 0; \tag{26}$$

$$2[T_{s1}(0, \tau) - f_1(\tau)] + \alpha_0 \left[\frac{\lambda_g \dfrac{\partial \varphi(0, \tau)}{\partial x}}{\rho v \left(C_{pg} + \dfrac{\partial C_{pg}}{\partial T_g} T_g(0, \tau) \right)} - \psi_1(0, \tau) \right] + \lambda_s \frac{\partial \psi_1(0, \tau)}{\partial x} = 0; \tag{27}$$

$$2[T_{sn-1}(b, \tau) - f_n(\tau)] = \lambda_s \frac{\partial \psi_{n-1}(b, \tau)}{\partial x}; \tag{28}$$

$$\varphi(0, \tau) = 0; \tag{29}$$

$$\varphi(b, \tau) = \frac{\partial \varphi(b, \tau)}{\partial x} = 0; \tag{30}$$

$$2[T_{si}(x_i, \tau) - f_i(\tau)] = \lambda_s \left[\frac{\partial \psi_{i-1}(x_i, \tau)}{\partial x} - \frac{\partial \psi_i(x_i, \tau)}{\partial x} \right], \; i = 2, 3, \ldots, n-1; \tag{31}$$

$$\psi_{i-1}(x_i, \tau) = \psi_i(x_i, \tau). \tag{32}$$

Then, taking account of (31), (27), and (21), we obtain for the linear part of the increment of the functional:

$$\Delta J = \int_0^{\tau_m} \Bigg\{ \sum_{i=2}^{n} \lambda_s \left[\frac{\partial \psi_{i-1}(x_i, \tau)}{\partial x} - \frac{\partial \psi_i(x_i, \tau)}{\partial x} \right] z_i(x_i, \tau) + \lambda_s(b, \tau) \frac{\partial \psi_{n-1}(b, \tau)}{\partial x} z_{n-1}(b, \tau) - \lambda_s z_1(0, \tau) \frac{\partial \psi_1(0, \tau)}{\partial x} -$$

$$- \alpha_0 z_1(0, \tau) \left[\frac{\lambda_g \dfrac{\partial \varphi(0, \tau)}{\partial x}}{\rho v \left(C_{pg} + \dfrac{\partial C_{pg}}{\partial T_g} T_g(0, \tau) \right)} - \psi_1(0, \tau) \right] \Bigg\} d\tau =$$

$$= \int_0^{\tau_m} \Bigg\{ \sum_{i=1}^{n-1} \int_{x_i}^{x_{i+1}} \left[\lambda_s \frac{\partial z_i}{\partial x} \frac{\partial \psi_i}{\partial x} + z_i \frac{\partial}{\partial x} \left(\lambda_s \frac{\partial \psi_i}{\partial x} \right) \right] dx +$$

$$+ \alpha_0 z_1(0, \tau) \left[\psi_1(0, \tau) - \lambda_g \frac{\partial \varphi(0, \tau)}{\partial x} \Big/ \left(\rho v \left(C_{pg} + \frac{\partial C_{pg}}{\partial T_g} T_g(0, \tau) \right) \right) \right] \Bigg\} d\tau.$$

$$\sum_{i=1}^{n-1}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}} z_i \frac{\partial}{\partial x}\left(\lambda_s \frac{\partial \psi_i}{\partial x}\right) dx d\tau = \sum_{i=1}^{n-1}\int_0^{\tau_m} \psi_i \frac{\partial \lambda_s z_i}{\partial x}\Big|_{x_i}^{x_{i+1}} d\tau -$$

$$-\sum_{i=1}^{n-1}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}} \lambda_s \frac{\partial z_i}{\partial x}\frac{\partial \psi_i}{\partial x} dx d\tau + \sum_{i=1}^{n-1}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}} \left\{ u \frac{\psi_i}{1-\Pi}\left(\alpha_V - \frac{\partial \alpha_V}{\partial T_g}(T_{si}-T_g)\right) - \Delta\alpha_V \frac{\psi_i}{1-\Pi}(T_{si}-T_g) + \right.$$

$$\left. + \psi_i \frac{\partial}{\partial x}\left(\Delta\lambda_s \frac{\partial T_s}{\partial x}\right) - \frac{\alpha_V z_i \varphi}{\Pi} \right\} dx d\tau.$$

Thus

$$\Delta J = \sum_{i=1}^{n-1}\int_0^{\tau_m} \psi_i \frac{\partial \lambda_s z_i}{\partial x}\Big|_{x_i}^{x_{i+1}} d\tau + \int_0^{\tau_m}\sum_{i=1}^{n-1}\int_{x_i}^{x_{i+1}} \left\{ \frac{\psi_i}{1-\Pi}\left[u \left(\alpha_V - \right.\right.\right.$$

$$\left.\left.\left. - \frac{\partial \alpha_V}{\partial T_g}(T_{si}-T_g)\right) - \Delta\alpha_V (T_{si}-T_g)\right] + \psi_i \frac{\partial}{\partial x}\left(\Delta\lambda_s \frac{\partial T_s}{\partial x}\right) - \frac{\alpha_V z_i \varphi}{\Pi} \right\} dx d\tau + \int_0^{\tau_m} \alpha_0 z_1(0,\tau) \times$$

$$\times \left[\psi_1(0,\tau) - \frac{\lambda_g \dfrac{\partial \varphi(0,\tau)}{\partial x}}{\rho v \left(C_{pg} + \dfrac{\partial C_{pg}}{\partial T_g} T_g(0,\tau) \right)} \right] d\tau.$$

Transforming the resulting expression, taking account of the conditions (32) and (25), and integrating by parts, taking account of (14), (16), (26), (29), and (30), we find

$$\Delta J = \sum_{i=1}^{n-1}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}} \left\{ \psi_i \frac{\partial}{\partial x}\left(\Delta\lambda_s \frac{\partial T_s}{\partial x}\right) - \Delta\alpha_V (T_{si}-T_g) \times \right.$$

$$\left. \times \left(\frac{\psi_i}{1-\Pi} - \frac{\varphi}{\Pi} \right) \right\} dx d\tau + \int_0^{\tau_m} \psi_1(0,\tau) \frac{\partial T_s}{\partial x} \Delta\lambda_s d\tau - \int_0^{\tau_m} \psi_{n-1}(b,\tau) \frac{\partial T_s}{\partial x} \Delta\lambda_s d\tau. \quad (33)$$

Since $\Delta J = (J_u', \Delta u)_{L_2}$, taking account of the fact that $\alpha_V = \alpha_V(\tau)$, we obtain

$$\frac{\partial J}{\partial \alpha_V} = -\sum_{i=1}^{n-1}\int_{x_i}^{x_{i+1}} (T_{si}-T_g)\left(\frac{\psi_i}{1-\Pi} - \frac{\varphi}{\Pi} \right) dx. \quad (34)$$

The expression for the gradient of the functional $\partial J/\partial\lambda_s$ depends on the form of the approximation of the relation $\lambda_s = \lambda_s(T_s)$. The specificity of the process of heat propagation in the porous material consists in the fact that at high temperatures we must take account not only of the variation of the thermal conductivity λ_s of the solid skeleton with temperature but also of the additional heat transfer by radiation, which is reflected in the coefficient of effective thermal conductivity by the term $\lambda_R(T_s)$. Taking account of the fact that for small temperature differences between the radiating surfaces, the radiant heat flux is proportional to the third power of the temperature of the porous skeleton [11], we can approximate the temperature-dependent variation of the effective thermal conductivity of the porous structure by a third-degree polynomial:

$$\lambda_s = \sum_{j=0}^{3} \lambda_j T_s^j.$$

Here we will have

$$J'_{\lambda_j}=\sum_{i=1}^{n-1}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}}\psi_i\left[jT_{si}^{j-1}\left(\frac{\partial T_s}{\partial x}\right)^2+T_{si}^j\,\frac{\partial^2 T_{si}}{\partial x^2}\right]dx\,d\tau+$$

$$+\int_0^{\tau_m}\left[\psi_1(0,\ \tau)\,\frac{\partial T_{si}}{\partial x}\,T_{s_1}^j(0,\ \tau)-\psi_{n-1}(b,\ \tau)\,\frac{\partial T_{sn-1}}{\partial x}\,T_{sn-1}^j(b,\ \tau)\right]d\tau. \tag{35}$$

A broader class of functions, widely used in recent years to approximate functional relations, is represented by B-splines. Therefore, in seeking a function $\lambda_s(T_s)$, we used an approximation of the desired function by cubic B-splines, as was done in a number of studies, for example [13, 14], devoted to the determination of how the thermal conductivity varies with temperature:

$$\lambda_s(T_s)=\sum_{j=-1}^{m+1}\lambda_j B_j(T_s), \tag{36}$$

where the λ_j are the desired coefficients of the spline and the $B_j(T_s)$ are the B-splines.

In this case, for the components of the gradient R of the target functional we have

$$\begin{gathered}J'_{\lambda_j}=\frac{\partial J}{\partial\lambda_j}=\sum_{i=1}^{n-1}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}}\psi_i\left[\frac{dB_j(T_s)}{dT_{si}}\left(\frac{\partial T_{si}}{\partial x}\right)^2+B_j(T_s)\frac{\partial^2 T_{si}}{\partial x^2}\right]dx\,d\tau+\\+\int_0^{\tau_m}\left[\psi_1(0,\ \tau)\,\frac{\partial T_{si}}{\partial x}\,B_j(T_s)-\psi_{n-1}(b,\ \tau)\,\frac{\partial T_{si}}{\partial x}\,B_j(T_s)\right]d\tau.\end{gathered} \tag{37}$$

Knowing the values of the gradients of the target functional we construct the process of successive approximations on the basis of the method of conjugate gradients [13]. In this case the approximations are carried out by the formulas

$$p^{(s+1)}=p^{(s)}+\gamma^{(s)}G^{(s)}, \tag{38}$$

where $\gamma^{(s)}=\{\gamma_{\alpha_V}^{(s)},\ \gamma_{\lambda_s}^{(s)}\}$; $p=\{\alpha_V,\ \lambda_s\}$; $G^{(s)}=\{G_{\alpha_V}^{(s)},\ G_{\lambda_s}^{(s)}\}$; $G^{(s)}=-J'^{(s)}+\beta^{(s)}G^{(s-1)}$; $J'^{(s)}=\{J'^{(s)}_{\alpha_V},\ J'^{(s)}_{\lambda_s}\}$; $\gamma^{(s)}$ is the depth of descent; s is the number of the iteration.

The choice of different descent steps for each component of the gradient of the functional being minimized, i.e., the determination of the step vector from the condition that the target functional must be a minimum with respect to this vector at each iteration, according to [15], enables us to speed up the convergence of the iterative process and reduce the influence of the relation between the individual desired quantities upon the rate of convergence. The values of the descent steps are selected from the condition that the target functional (12) must be a minimum:

$$\min_{\gamma_1,\gamma_2>0}\left\{\sum_{i=1}^{n}\int_0^{\tau_m}[T_{si}(\alpha_V+\gamma_1\Delta\alpha_V,\ \lambda_s+\gamma_2\Delta\lambda_s)-f_i(\tau)]^2\,d\tau\right\}=$$

$$=\min_{\gamma_1,\gamma_2>0}\left\{\sum_{i=1}^{n}\int_0^{\tau_m}[T_{si}+\gamma_1\Delta T_{si}(\Delta\alpha_V)+\gamma_2\Delta T_{si}(\Delta\lambda_s)-f_i(\tau)]^2\,d\tau\right\}. \tag{39}$$

In using the given linear approximation for determining $\gamma^{(s)}$, we obtain a system of linear equations

$$\gamma_1\sum_{i=1}^{n}\int_0^{\tau_m}\Delta T_{si}^2(\Delta\alpha_V)\,d\tau+\gamma_2\sum_{i=1}^{n}\int_0^{\tau_m}\Delta T_{si}(\Delta\alpha_V)\,\Delta T_{si}(\Delta\lambda_s)\,d\tau=-\sum_{i=1}^{n}\int_0^{\tau_m}(T_{si}-f_i)\,\Delta T_{si}(\Delta\alpha_V)\,d\tau,$$

$$\gamma_1\sum_{i=1}^{n}\int_0^{\tau_m}\Delta T_{si}(\Delta\alpha_V)\,\Delta T_{si}(\Delta\lambda_s)\,d\tau+\gamma_2\sum_{i=1}^{n}\int_0^{\tau_m}\Delta T_{si}^2(\Delta\lambda_s)\,d\tau=-\sum_{i=1}^{n}\int_0^{\tau_m}(T_{si}-f_i)\,\Delta T_{si}(\Delta\lambda_s)\,d\tau. \tag{40}$$

The iterative process is constructed as follows. We are given the initial values of the desired parameters, we solve the direct boundary-value problem describing the process of heat exchange in the porous solid when the coolant is blown through, (1)-(8), and we determine the temperature fields. After this, using the solution of the conjugate boundary-value problem (24)-(30), we calculate the components of the gradient of the target functional. Next we solve twice the increment problem (13)-(20), and from the solution of the system of equations (40) we determine the values of the descent steps. We calculated the new values of the desired parameters by the formulas (38), and we repeat the calculation process. In setting up the algorithm, we assumed that if exact initial data are given, the iterative process is halted when the desired values in two successive iterations coincide, and if the input data are known with some error, the halt is made according to the discrepancy criterion, i.e., the number of the last iteration s* is chosen on the basis of the condition

$$s^*: \sum_{i=1}^{n} \int_{0}^{\tau_m} [T_{si}(x_i, \tau) - f_i(\tau)]^2 \, d\tau \simeq \delta^2,$$

where $\delta^2 = \sum_{i=1}^{n} \int_{0}^{\tau_m} \sigma_i d\tau$ is the estimate of the generalized error of the initial data; $\sigma_i(\tau)$ is the mean-square deviation of the input temperatures at the points $x = x_i$ at time τ.

The above-described algorithm formed the basis for the program set up in Fortran for the BÉSM-6 computer; the differential equations (1), (2), (13), (14), (24), and (25) were approximated by means of a monotonic implicit difference scheme of second-order accuracy [16]. The direct heat-exchange problem in the porous solid was nonlinear, and therefore in solving it, at each time step, we carried out iterations with respect to the coefficients of the equations, coming out of the iterative process by the condition that the temperature profiles at two successive approximations had to coincide to a given degree of accuracy.

The operational capability of the above algorithm was verified by using model examples in the reconstruction of the coefficient of internal heat exchange in a porous solid. Special attention was given to the possibility of solving the problem by using measurements made only on the outer and inner surfaces, since making temperature measurements inside a porous solid is very difficult and often is impossible without disturbing the nature of the coolant flow within the porous structure. Thus, the maximum number of measurements made is two. Therefore in performing our model calculations, we assumed that the temperature dependence of the effective thermal conductivity is given exactly or is determined in advance by an experiment which does not include blowing of the coolant (for $\rho v = 0$ the problem of determining λ_s is analogous to the problem given in [13], but with boundary conditions of the second kind).

It was also assumed that a porous solid 5 mm thick has the following characteristics: the variation with temperature, $\lambda_s(T_s)$, is described by the formula

$$\lambda_s = 0.1638 - 0.256 \cdot 10^{-4} \, T_s + 5.8 \cdot 10^{-8} \, T_s^3, \ \mathrm{kW/m \cdot K};$$

the volumetric heat capacity

$$C_s = 1.2155 - 0.984\,(T_s - 273) + 0.00234\,(T_s - 273)^2, \ \mathrm{kJ/m^3 \cdot K};$$

the viscous and inertial coefficients of resistance were taken to be

$$\alpha = 3.18 \cdot 10^{11}, \ \ 1/\mathrm{m}^2, \ \ \beta = 8.5 \cdot 10^{6}, \ \ 1/\mathrm{m}.$$

The gas blown through the apparatus was air with a constant input temperature of $T_{g_0} = 300$°K. The coefficient of heat exchange at the input was taken to be zero. The heat flux impinging on the outer surface of the porous solid was given by the formula

$$q_w = 1000 \left(\frac{\tau}{\tau_m} + \sin \pi \frac{\tau}{\tau_m} \right), \ \ \frac{\mathrm{kW}}{\mathrm{m}^2}.$$

The rate of air blowing was taken to be variable: $\rho v = \tau/100$ for $0 \leqslant \tau \leqslant \tau_m/2$, and $\rho v = 0.12 - (\tau - \tau_m/2)/100$ for $\tau > \tau_m/2$. The duration τ_m of the experiment was 24 sec.

TABLE 1. Values of the Desired Parameters and the Functional Obtained by Iterations (the exact values of the parameters are A = 4 and n = 4)

Number of model example	Coordinates of the thermocouples	Initial approximation	Iterations						
			1st	2nd	3rd	4th	5th	6th	7th
1	$x_1=0$, $x_2=,004$, $x_3=,0045$	$A=1$, $n=1$, $F=1240$	1,365 2,037 167,5	2,169 3,143 28,03	2,987 3,682 3,582	3,278 3,757 0,194	3,468 3,859 1,920	3,589 3,857 0,430	3,733 3,944 1,270
2	$x_1=0$, $x_2=,005$	$A=1$, $n=1$, $F=430,7$	1,352 1,942 73,20	2,079 2,960 3,065	2,936 3,617 0,632	3,343 3,789 0,049	3,492 3,849 0,247	3,652 3,893 0,0078	3,747 3,9333 0,1295
3	$x_1=0$ $x_2=,005$	$A=1$, $n=1$, $F=430,7$	1,234 1,628 145,0	1,654 2,320 40,22	2,226 2,958 7,192	2,820 3,430 1,205	3,262 3,685 0,1587	3,529 3,817 0,0294	3,672 3,879 0,01668
4	$x_1=,005$	$A=1$, $n=1$, $F=186,6$	1,244 1,640 62,476	1,667 2,319 19,77	2,215 2,930 4,075	2,796 3,388 0,799	3,239 3,660 0,117	3,517 3,803 0,0185	3,664 3,869 0,0055

TABLE 2. Relative Values of the Functional Obtained by Different Placement of the Thermocouples

Coordinates of the thermocouples	Iterations							
	1st	2nd	3rd	4th	5th	6th	7th	8th
$\bar{x}=0$	0,304	0,0786	0,0128	0,001825	$2,67\cdot10^{-4}$	$9,95\cdot10^{-5}$	$6,35\cdot10^{-5}$	$5,74\cdot10^{-5}$
$\bar{x}=1$	0,335	0,106	0,0207	0,00429	$6,2\cdot10^{-4}$	$9,9\cdot10^{-5}$	$2,96\cdot10^{-5}$	$1,346\cdot10^{-5}$
$\bar{x}=0$ $\bar{x}=1$	0,337	0,0935	0,0167	0,0028	$3,69\cdot10^{-4}$	$6,84\cdot10^{-5}$	$3,88\cdot10^{-5}$	$2,11\cdot10^{-5}$

The values of the coefficient of internal heat exchange were determined from the expression

$$\alpha_V = A\,\frac{\lambda_g}{(\beta/\alpha)^2}\left(\frac{\beta/\alpha\rho v}{\mu_g}\right)^n, \tag{41}$$

where the coefficient A and the exponent n took on different values.

There are two possible ways to reconstruct the coefficient of internal heat exchange: either searching for a large number of discrete values of α_V that enabled us to minimize the functional (12) with a given accuracy or parametrizing the problem, i.e., searching for values of α_V in the form of relations of the type (41) (the derivation of these is usually the last step in the processing of experimental data [1-7]), when the values of A and n are the desired parameters.

As was shown by the results of the numerical modeling, when the first method is used, the iterative process is less sensitive to perturbations in the input data. Among the disadvantages of the first method is the need to obtain a subsequent approximation to the results in the form of relations of the type (41), which adds to the error in the final result. Therefore attention was given primarily to the second method of determining the coefficient of internal heat exchange.

Table 1 shows the results of the numerical modeling which were obtained with exact input data and which illustrate the convergence of the iterative process. As can be seen from the results shown (model examples 1 and 2), the linear estimate of the depth of a step for the conjugate gradient, as obtained by solving the system of equations (40), does not guarantee monotonic convergence. It may be expected that taking account of the third term in the Taylor expansion will enable us to improve this:

$$\min_{\gamma_1,\gamma_2>0}\left\{\sum_{i=1}^{n}\int_0^{\tau_m}[T_{si}(\alpha_V+\gamma_1\Delta\alpha_V,\ \lambda_s+\gamma_2\Delta\lambda_s)-f_i(\tau)]^2\,\partial\tau\right\}=$$
$$=\min_{\gamma_1,\gamma_2>0}\left\{\sum_{i=1}^{n}\int_0^{\tau_m}\left[T_{si}+\gamma_1\frac{\partial T_{si}}{\partial\alpha_V}\Delta\alpha_V+\frac{1}{2}\frac{\partial^2 T_{si}}{\partial\alpha_V^2}\gamma_1^2\Delta\alpha_V^2+\gamma_2\Delta\lambda_s\frac{\partial T_{si}}{\partial\lambda_s}+\frac{1}{2}\frac{\partial^2 T_{si}}{\partial\lambda_s^2}\gamma_2^2\Delta\lambda_s^2-f_i(\tau)\right]^2 d\tau\right\}. \tag{42}$$

We use the following approximation method. Taking account of the fact that

$$\Delta T_{si}(\Delta p_j)\simeq\frac{\partial T_{si}}{\partial p_j}\Delta p_j \text{ and } \frac{\partial^2 T_{si}}{\partial p_j^2}\simeq\left(\left(\frac{\partial T_{si}}{\partial p_j}\right)-\left(\frac{\partial T_{si}}{\partial p_j}\right)'\right)/\Delta p_j,$$

we write

$$\frac{\partial^2 T_{si}}{\partial p_j^2}\Delta p_j^2\simeq\frac{\Delta T_s(\Delta p_j)-\Delta T_s(\Delta p_j')}{\Delta p_j-\Delta p_j'}\Delta p_j,$$

and we obtain the following system of equations for determining the depth of the step:

$$\frac{\partial J}{\partial\gamma_k}=\sum_{i=1}^{n}\int_0^{\tau_m}\left[T_{si}+\sum_{j=1}^{2}\left(\Delta T_{sij}+\frac{1}{2}\frac{\Delta T_{sij}-\Delta T_{sij}'}{\Delta p_j-\Delta p_j'}\Delta p_j\right)\gamma_j-f_i\right]\times$$
$$\times\left(\Delta T_{sik}+\frac{1}{2}\frac{\Delta T_{sik}-\Delta T_{sik}'}{\Delta p_k-\Delta p_k'}\Delta p_k\right)d\tau=0,\quad k=1,\ 2. \tag{43}$$

The values $\Delta p'_j$ are best chosen to be $\Delta p_j/2$; then the system (43) becomes

$$\sum_{i=1}^{n}\int_0^{\tau_m}\left[T_{si}+\sum_{j=1}^{2}\gamma_j(2\Delta T_{sij}-\Delta T_{sij}')-f_i\right](2\Delta T_{sik}-\Delta T_{sik}')\,d\tau=0,$$
$$k=1,\ 2. \tag{44}$$

The choice of the γ_j from the conditions (44) enables us to guarantee monotonic convergence of the iterative process (examples 3 and 4 in Table 1).

Table 2 shows the influence of thermocouple placement on the convergence. It can be seen that placing a thermocouple on the outer heated surface $\bar{x} = 1$ is preferable to placing it on the inner surface, since it enables us to obtain a lower value of the functional after the same number of iterations.

In the course of real experiments the temperature measurements always involve some error of a random or systematic nature; therefore, in order to verify the operational capability of the above algorithm under real conditions on the temperature values that play the role of measurements, during some model calculations we imposed random perturbations which followed a normal distribution law. It was found that in order to solve the problem with perturbed data, we first had to edit the real measurements, for example by smoothing the experimental data, or plan the experiments with a view to indicating the regimes of maximum sensitivity of the functional to the desired parameters. The reason for this is that in some regimes, after we have selected the initial approximation for the desired parameters, the values obtained by solving the direct problem (1)-(8) for the temperatures at the points of fixation of the thermocouples will immediately fall in the tube of admissible values, and the determination of the direction of descent therefore has a more or less random character.

On the whole, the mathematical verification showed that the above-described approach is effective in determining the coefficients of heat exchange in a porous solid.

NOTATION

x, coordinate; b, thickness of the porous body; n, number of measurements of the temperature of the porous body; C_s, λ_s, volumetric heat capacity and effective thermal conductivity of the porous body; ρ, C_p, λ, density, specific heat, and thermal conductivity of the blown gas; T, temperature of the porous skeleton and the gas; α_V, coefficient of internal a heat exchange; ρv, rate of blowing; τ, time; τ_m, duration of experiment; p, pressure; M, molecular weight of the gas; α, β, coefficients of hydraulic resistance of the porous body; ψ, φ, conjugate variables; q_w, heat flux into the wall at the outer boundary; Π, porosity; α_0, coefficient of heat exchange at the inlet to the porous body; q_V, intensity of internal heat generation; s, g, indices corresponding to the solid and gaseous phases; μ, viscosity.

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